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Mathematical Stagnation

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Review Article

Mathematical Stagnation

IT is now generally recognized by respected pedagogues that mathematics is most effectively taught, whether to a theoretical or practical man, in terms of concepts and ideas rather than rules and techniques. At a time when publishers the world over are busily engaged in the production of more and better textbooks of this type, it is indeed sad to note the appearance of several introductory mathematical textbooks bearing the authority of some of the better technological schools in the Philippines which are diametrically opposed to this ideal. This feeling is especially heightened by the knowledge that there are already available on these same topics textbooks of better quality and less expensive. One can only wonder whether this is indicative of the sorry state of technological education in this country.

There is nothing novel about the textbooks under review. In fact, both of them tenaciously cling to a weak and obsolete tradition. Like many of their older predecessors, they imitate the blunders and weaknesses of a long line of equally bad texts in algebra and trigonometry.

In the very words of the authors of *Plane and Spherical Trigonometry*:

> This book was prepared in response to a number of requests from instructors in mathematics for a textbook in trigonometry that will be

within the level of comprehension of students starting their first year in college.[1]

With equal emphasis *College Algebra* states:

To fill a great need for a book best adapted to local conditions, this book on college algebra was prepared.[2]

Considering the existence, as has been said, of local textbooks by Vidal Tan and Francisco Perez on the same subjects, these statements can only mean that Doctors Tan and Perez's books are incomprehensible to students. This is perhaps true, not because our students are stupid, but because of the very poor quality of our high-school and college teachers in mathematics. Nowadays, it does not really require much to become a so-called professor of mathematics: one has simply to finish the course. It is not unlikely that the books of Tan and Perez have lost their appeal because they are incomprehensible to many instructors and so-called professors themselves.

As a matter of fact, Dr. Francisco Perez himself once opined that the main reason why his textbooks are losing popularity in our schools is the fact that many instructors and so-called professors of mathematics find some of his miscellaneous problems extremely difficult. These problems have been the cause of many embarrassing moments for professors in the classroom.

Now, is it justified to dilute a course or a textbook merely because our instructors and so-called professors of mathematics are unqualified? How really sound is such an educational policy? One ought to remember that elementary algebra and plane trigonometry were once only high-school courses; in fact, they still are in some schools. Their apperance in the college curriculum was made necessary, so they say, by the pressure of weaker and weaker students entering college; but in reality more because of weaker and weaker teachers of mathematics.

[1] PLANE AND SPHERICAL TRIGONOMETRY. By a Committee On Mathematics. The Philippines: Filipino Educators, Inc., 1962. ix, 162 p. Tables. ₱10.80.

[2] COLLEGE ALGEBRA. By A Committee On Mathematics. Quezon City, Philippines: Filipino Educators,, Inc. 1962. xiv, 263 p. ₱9.60.

If we yield to such continued pressure, then one can safely predict that in the near future arithmetic and plane geometry will have to be taught in college or in the university rather than in grade school and high school.

As far as mathematical books go, a very rare feature of the books under consideration is their copiousness in exposition, instructions and rules to follow. Remarks that would ordinarily be awkward and inelegant to mention in print are explicity asserted in the books. Not too much room is left to the intelligence of the student and the individuality of the professor. Their paucity of thought-provoking passages reminds the reviewer very much of a recipe book. By blindly following their instructions one concocts a dish without ever knowing why it should be such.

The reviewer has sought the pages of the trigonometry text for the realization of the author's fundamental objective, "the development of sound mathematical reasoning and the applications of the various principles and formulas in the fields of science and technology." Trigonometry is hardly the course where sound mathematical reasoning can be developed. This is a fact generally accepted in mathematical circles. The utilitatarian value of trigonometry in modern technology does not depend on the fact of its having anything to do with triangles but rather on its special functional aspect. It may be that trigonometric functions have their origins in triangles and are still important to geodesists and surveyors, but modern science and technology have found more powerful tools by thinking of them as periodic curves, as solutions of differential equations, or as approximations of many other functions. This so-called *functional* approach which has characterized recent textbooks in trigonometry is noticeably neglected in this book.

For the sake of illustration, let us pick out a few isolated pronouncements of the textbook. "We define function as a quantity whose value depends on the value of another quantity." "It is helpful to think of a trigonometric function as a fraction." "All angles irrespective of magnitude and sign have trigonometric functions." "The functions are constant for any angle". "Tables of Natural Functions." These and many

others lead the reviewer to believe that the authors themselves are groping in the dark for the precise meaning of a function. Undoubtedly, they have in mind the definition of Dirichlet: *a (single-valued) function of a variable x is a second variable y so related to x that whenever a value is assigned to x, a corresponding value of y is determined.* However, they seem to have failed to realize the significance of the definition when they confused the value of a function with the function itself. Dirichlet's definition is vague enough; such a misconception only makes things worse than they really are. No wonder many of our students go to graduate school without ever fathoming the mystery of a function.

A function, of course, is most precisely defined as a set of ordered pairs, but this may be too sophisticated. The best compromise, I believe, is to think of a function as a matching of the elements of two collections whereby each element of the first collection is matched uniquely with an element of the second. The function denoted by $f(x) = x^2$ is then the function which matches with each real number its square. A trigonometric function is one that matches each angle with a real number. Thus, for instance, the sine function is one that matches each angle with a fixed number between -1 and 1.

Comparatively speaking, the book on college algebra is of better quality than the one on trigonometry, but like the first it suffers from some conceptual difficulties. The "conceptual vigor" which the authors so confidently talk about is centuries-old and in state of what may be called *vigor mortis*. It presents, for instance, some of the fundamental laws of operations for real numbers without ever motivating its necessity in the study of algebra. Consequently, notwithstanding the authors' cryptic remark that they are important in extending the natural numbers to the reals, these laws appear to the poor student as a collection of obvious and trivial properties that he already knows.

Such statements as "in the generalization of symbols used in algebra the natural number system of arithmetic has to be extended at least to the real number system" are simply misleading. Mathematical symbols by themselves are meaningless

and can never be therefore generalized. The symbol x and even the statement-form *x is an integer* are meaningless. They are not mathematical propositions for they are neither true nor false. While perhaps it is true that one cannot completely get rid of the wrong practice in elementary mathematics of classifying quantities into variables and constants, textbook writers, I believe, must avoid stating explicitly wrong ideas. Or, if one, by reason of expediency, cannot really avoid conceptual corruptions, he should explicitly delineate them. It is encouraging to know that more and more books in college algebra abroad are pursuing this conceptual sanity and sensitivity. It is far better to teach a few things that are right than to teach many things that are wrong. The process of unlearning is often more difficult than the process of learning.

While our comments have been severe we hope that they will be taken as they are intended, as constructive criticism which may help in the production of better mathematical texts by Filipino authors.

F. M. Sioson